Experimental Validation of Robotic Manifold Tracking in Gyre-Like Flows

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***Abstract*— In this paper, we present a first attempt toward experimental validation of a multi-robot strategy for tracking manifolds and Lagrangian coherent structures (LCS) in flows. LCS exist in natural fluid flows at various scales, and they are time-varying extensions of stable and unstable manifolds of time invariant dynamical systems. In this work, we present the first steps toward experimentally validating our previously proposed real-time manifold and LCS tracking strategy that relies solely on local measurements. Although we have validated the strategy in simulations using analytical flow models, experimental flow data, and actual ocean data, the strategy has never been implemented on an actual robotic platform. We demonstrate the tracking strategy using a team of micro autonomous surface vehicles (mASVs) in our laboratory testbed and investigate the feasibility of the strategy with vehicles operating in an actual fluid environment. Our experimental results show that the team of mASVs can successfully track LCS using a simulated velocity field, and we present preliminary results showing the feasibility of a team of mASVs tracking manifolds in real flows using only local measurements obtained from their onboard flow sensors.**

I. INTRODUCTION

We are interested in the development of collaborative control strategies for distributed sensing and tracking of coherent structures and manifolds in flows using teams of autonomous underwater vehicles (AUVs). Coherent structures are important because they give us insight into the dynamics of the surrounding fluidic environment. In recent years, researchers have shown that AUV motion planning and adaptive sampling strategies can be improved by incorporating either historical ocean flow data [1]–[3] or multi-layer partial differential equation (PDE) models of the ocean [4], [5]. However, accessibility to and the overall quality of the flow data and/or numerical models is highly dependent on how well a given region of interest is instrumented. This is because numerical PDE models are often derived through a combination of theoretical and field observations, and ocean current hindcasts, nowcasts, and forecasts provided by Navy Coastal Ocean Model (NCOM) databases [6] and regional ocean model systems (ROMS) [2] are assimilated from satellite and field observations in conjunction with predictions from numerical PDE models [7], [8]. As such, despite various public and private organizations' efforts in the last thirty to forty years to deploy a combination of stationary, surface, and at-depth sampling technologies, existing data sets that describe ocean flows are still mostly finite-time and of low spatio-temporal resolution.

Geophysical fluid dynamics (GFD) is the study of natural large-scale fluid flows, such as oceans, eddies, and rivers. While GFD flows are naturally stochastic and aperiodic, they do exhibit coherent structures. Recently, Inanc et al. showed that time and fuel optimal paths in the ocean can coincide with a specific class of coherent structures called Lagrangian coherent structures (LCS) [9], [10]. LCS are the extensions of stable and unstable manifolds to general time-dependent flows [11] and are similar to separatrices that divide the flow into dynamically distinct regions. For two-dimensional (2D) flows, LCS are analogous to ridges defined by local maximum instability, and can be quantified by local measures of Finite-Time Lyapunov Exponents (FTLE) [12]. Since LCS are inherently unstable and denote regions of the flow where more escape events may occur [13], knowledge of LCS locations is also important for maintaining sensors in specific monitoring regions.

In our previous work [14], we developed a collaborative robotic control strategy for tracking stable and unstable manifolds in 2D flows. The technique relies on robots performing local measurements of the flow field and fusing this information to collaboratively track these boundaries. While the proposed strategy has been validated using the scale invariant analytical wind-driven double-gyre flow model often used to describe large scale ocean circulation, experimental data generated by a flow tank, and actual ocean data, the strategy has never been validated using an actual robotic platform. Furthermore, since the distributed estimation of the LCS boundary locations depends on a variety of factors such as sensor and actuation noise, the sampling frequency, and the time scales of the flow dynamics, proper validation of the tracking strategy described in [15] must be performed using different ocean data sets. However, given the low spatio-temporal resolution of existing GFD data sets and the need to know the locations of the LCS boundaries *a priori*, this makes evaluation of any existing and future tracking strategies extremely challenging.

In this work, we present a first attempt towards the experimental evaluation of collaborative strategies for mobile robot teams tracking LCS boundaries in 2D flows using real robots in a controllable laboratory setting. Specifically, we

*This work was supported by the Office of Naval Research (ONR). MM and MAH are supported by ONR Contract No. N0001411WX20079. IBS is supported by ONR Contract No. N0001412WX20083 and NRL Base Funding Contract No. N0001414WX00023.

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describe the development of an appropriate indoor multi-robot coherent structure tracking testbed, the experimental methodology employed, and the experimental results obtained in both time-invariant and time-dependent 2D flows. The novelty of our contribution lies in the experimental validation of our existing LCS tracking strategy using a team of micro-autonomous surface vehicles (mASVs) subject to ocean-like flows.

The paper is structured as follows: Section II provides some background and briefly summarizes the collaborative tracking strategy. We describe our experimental methodology in Section III and present our experimental results in Section IV. We conclude with a brief discussion of our results in Section V and a summary of ongoing and future work in Section VI.

II. BACKGROUND

As mentioned previously, the existing robotic manifold and LCS tracking strategy was validated using scale invariant analytical models that are time-independent and time-varying [16], experimental flow data created in a flow tank [15], and actual ocean data [14]. The objective of this work is to experimentally validate the strategy described using an actual robotic platform. We briefly summarize our tracking strategy for the sake of completeness and refer the interested reader to [15] for further details.

The collaborative control strategy was developed assuming the following 2D kinematic model for each AUV:

$$\dot{x}_i = V_i \cos\theta_i + u_i, \tag{1a}$$

$$\dot{y}_i = V_i \sin\theta_i + v_i, \tag{1b}$$

where $\mathbf{x}_i = [x_i, y_i]^T$ denotes the vehicle's position in the plane, V_i and θ_i denote respectively the vehicle's forward speed and heading, and $\mathbf{u}_i = [u_i, v_i]^T$ denotes the velocity of the fluid measured by the i^{th} vehicle. In this work, we assume V_i and θ_i are the control inputs for each vehicle. The objective is for the robot team to maintain a valid *saddle straddling formation* across the manifold/boundary of interest at all times to enable the team to iteratively track the boundary location in the flow.

Given a team of three robots, each denoted as $\{L,C,R\}$, robot C is tasked to remain close to the stable manifold of interest, denoted by B_S. Robots L and R are tasked to remain on opposite sides of B_S at all times and thus maintain the saddle straddle formation at all times. This is achieved by leveraging the flow field dynamics and estimating the location of B_S using only local measurements of the velocity field. The controller for the straddling robots consists of two discrete states: a passive control state, U_P, and an active control state, U_A. The robots initialize in the passive state U_P where the objective is to follow the flow of the ambient vector field. Therefore, $V_i = 0$ for $i = L,R$. Robots execute U_P until they reach a maximum allowable separation distance from the tracking robot C. When robots L and R are too far from robot C, they switch to the active control state, U_A, where the objective is to navigate to a point on the next saddle straddle line segment. A sketch of the collaborative control strategy is shown in Fig. 1.

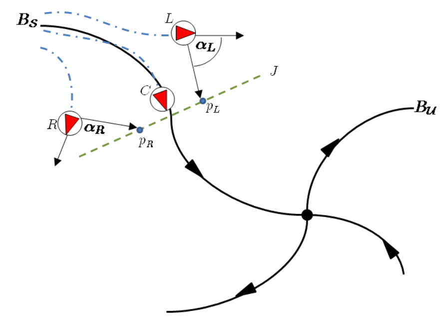


Fig. 1. Three robots tracking B_S in a given conservative vector field. The robot trajectories are denoted by blue dash-dot curves, the saddle straddle line segment J is shown by the green dashed line, and $\mathbf{p}_L$ and $\mathbf{p}_R$ denote the target positions for robots L and R, respectively, when executing U_A.

As the robots move through the flow in formation, they sample the velocity of the surrounding flow field and communicate their measurements and their relative positions to robot C. Robot C uses the flow velocity measurements obtained by robots L and R to interpolate the vector field along a collection of points located on the saddle straddling line segment, denoted by J in Fig. 1. The saddle straddling line segment J is defined by the positions of robots L and R. Robot C's estimate of the manifold location is given by the point on J with either the maximum or minimum velocity depending on whether the team is tracking the unstable or stable manifold. In this work, we assume vehicles operate in 2D conservative planar flows, know the positions of the other vehicles in relation to itself, and have full communication capability with the rest of the team. As such, robots L and R can communicate their measurements to robot C. Robot C can fuse this information to determine the boundary location and can communicate information about the projected saddle straddle line segment to robots L and R. By employing this technique iteratively, the team can estimate and track the location of the manifold.

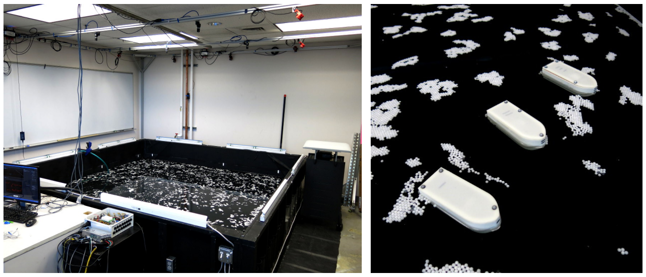
We remind the reader that LCS are time-dependent extensions of the stable and unstable manifolds of time-independent systems. And while the strategy was developed assuming robots operate in planar conservative flow fields, validation of the strategy using time independent and time-varying analytical flow models, experimental flow tank data, and actual ocean data [15] showed success, even for non-conservative flow fields.

III. METHODOLOGY

To evaluate the tracking strategy using experimental robotic platforms, we employ our multi-robot Coherent Structure Testbed (mCoSTe) [17].

A. Experimental Setup

The mCoSTe is an indoor laboratory experimental testbed that consists of three flow tanks and a fleet of two types of micro-autonomous surface vehicles: the mASV and the mASV*f*. The mASVs are differential drive surface vehicles



(a) (b)

Fig. 2. Photos of (a) multi-robot (MR) tank and (b) three mASVs in the MR tank. Visible in (b) are the unique patterns of retro-reflective markers for overhead motion capture tracking.

equipped with a micro-controller board, XBee radio module, and an inertial measurement unit (IMU). The vehicles are approximately 12 cm long and have a mass of about 45 g each. The mASV*f*s are similar to the mASVs except they equipped with onboard flow sensors capable of measuring the relative speed and direction of the local flow field. Each mASV*f* is approximately 15 cm long and has a mass of about 110 g. Localization for both the mASVs and the mASV*f*s is provided by an external motion capture system.

In addition to the vehicles, we also employed two of the mCoSTe's experimental flow tanks: the High Reynolds number (HiRe) Tank and the Multi-Robot (MR) Tank, which are respectively $0.6 \times 0.6 \times 0.3$ m^3 and $3 \times 3 \times 1$ m^3 in size. The MR tank is pictured in Fig. 2(a). Flow fields generated in the HiRe tank are extracted using particle imaging velocimetry (PIV). Both the HiRe and MR tanks were designed to be able to create time-independent and time-varying flow fields that exhibit kinematic and transport features similar to those observed in the ocean. The flows in the tanks are patterned after the wind-driven double gyre flow model given by:

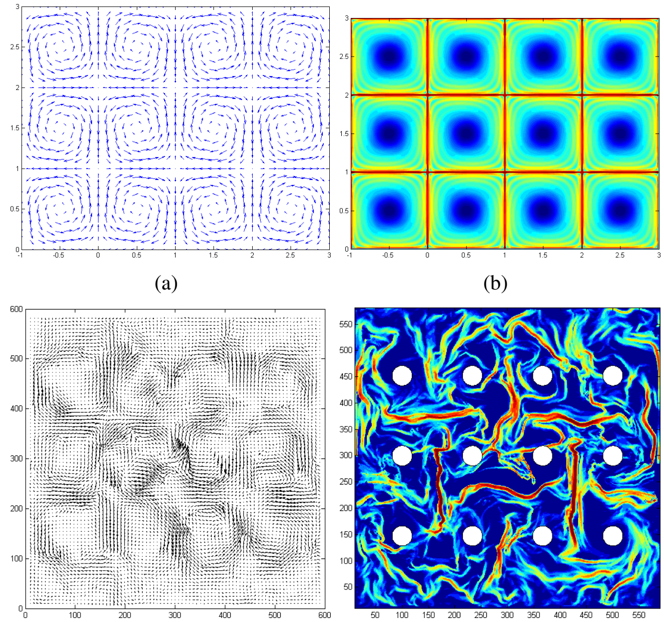
$$u = -\pi A \sin(\pi \frac{f(x,t)}{s}) \cos(\pi \frac{y}{s}) - \mu x, \tag{2a}$$

$$v = \pi A \cos(\pi \frac{f(x,t)}{s}) \sin(\pi \frac{y}{s}) \frac{df}{dx} - \mu y, \tag{2b}$$

$$f(x,t) = \varepsilon \sin(\omega t + \psi) x^2 + (1 - 2\varepsilon \sin(\omega t + \psi)) x \tag{2c}$$

which is often used to describe large scale recirculation in the ocean [18]. In Eqn.(2), when $\varepsilon = 0$, the flow is time-independent, while for $\varepsilon \neq 0$, the gyres undergo a periodic expansion and contraction in the x direction. Additionally, A approximately determines the amplitude of the velocity vectors, $\omega/2\pi$ gives the oscillation frequency, ε determines the amplitude of the left-right motion of the separatrix between the gyres, ψ is the phase, μ determines the dissipation, and s scales the dimensions of the workspace.

We recently showed that the flows created in the HiRe tank show good correspondence with the analytical model given by (2) [15], [17]. Fig. 3(a) shows the phase portrait for a grid of 3×4 gyres given by (2). The corresponding FTLE field is shown in Fig. 3(b). The manifolds/LCS correspond to regions with maximum FTLE measures which are shown in red. A snapshot of the surface flow measured using PIV



(a) (b) (c) (d)

Fig. 3. (a) Phase portrait for a grid of 3×4 gyres given by (2). (b) Corresponding FTLE field at $t = 0$ for (a) using an integration time of 1.25 sec. (c) Snapshot of surface flow data from the HiRe tank obtained via PIV. (d) Corresponding FTLE field at $t = 0$ for (c) using an integration time of 8 sec. [17]

in the HiRe tank for a similar arrangement of gyres is shown in Fig. 3(c). The HiRe flow was created using a 3×4 grid of flow driving cylinders. By setting the rotational speed and directions of the cylinders, we can create surface flows similar to the analytical model. The data shown in Fig. 3(c) was generated to match the time-invariant model. Fig. 3(d) shows the computed FTLE field for the corresponding HiRe flow data. Gyre-like flows patterned after (2) can also be created in the MR tank using a similar flow driving mechanism.

B. Experimental Methodology

To evaluate the feasibility of the proposed LCS tracking strategy using the mCoSTe, we deployed a team of 3 mASVs/mASV*f*s in the MR tank. Fig. 2(b) shows 3 mASVs in the MR tank. We considered the following flow fields in our experimental evaluation.

1) Case 1: A *time-invariant* flow field where $\mathbf{u}_i$ is given by the time invariant wind driven double-gyre model given by (2) with $A = 0.2$, $\varepsilon = 0$, $\mu = 0$, and $s = 1.5$. In this case the mASVs were deployed in the MR tank in still water.

2) Case 2a: Similar to Case 1, but the flow field is *periodic* (instead of time invariant), with $\mathbf{u}_i$ given by (2) with $A = 0.2$, $\varepsilon = 0.1$, $\mu = 0.005$, $\omega = 0.05$, $\psi = 0$, and $s = 1.5$. Again, the mASVs were deployed in the MR tank in still water.

3) Case 2b: Similar to Case 2a but zero-mean Gaussian noise was added to each $\mathbf{u}_i$ to simulate sensor measurement noise on a real robotic vehicle. Again, the mASVs were deployed in the MR tank in still water.

4) Case 3: Actual flow data obtained from the HiRe tank. A time-invariant flow field was created in the HiRe tank using a grid of 3×4 flow driving cylinders. The data was collected, scaled, and played back during the manifold/LCS tracking experiment. As such, the flows "experienced" and "measured" by the mASVs were actual flows, however, the flows were created within the HiRe tank. It is important to note that while the flow field created in the HiRe tank was modeled after the time-invariant double-gyre model, the flow data is quite noisy, as evidenced by Figs. 3(c) and 3(d). Again, the mASVs were deployed in the MR tank in still water.

5) Case 4: A time-invariant flow field was created using a 2×2 grid of flow driving cylinders in the MR tank. A team of 2 mASV*f*s were deployed such that the two vehicles straddle a manifold. Each vehicles obtains local measurements of $\mathbf{u}_i$ using their onboard flow sensors. Different from the previous cases, the mASV*f*s operate in the MR tank within a *real* flow field.

IV. RESULTS

We briefly summarize our experimental results for the cases enumerated above. We refer the reader to the multimedia attachment to this paper for videos of the various cases[1].

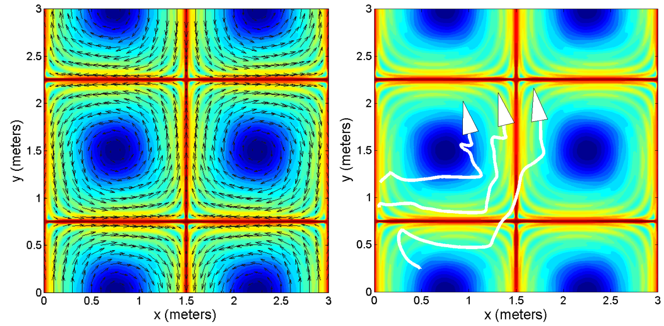
A. Case 1

In this case, the experiment was performed in still water in the MR tank where the robots' flow sensor outputs were simulated, i.e., $\mathbf{u}_i$ is given by the time-invariant wind drive double-gyre model described by (2). Even though there were no actual flows in the MR tank during these experiments, the motion of the mASV team was enough to generate a background flow in the MR tank which can impact the tracking strategy, making this scenario more realistic than pure simulations. Fig. 4(b) shows the trajectories (obtained using an external motion capture system) of 3 mASVs in the MR tank. It can be seen that the robots remain in a straddling formation across the manifold, shown in red, until reaching a hyperbolic point in the flow. At this point, the team shifts to track another manifold in the flow. This behavior is desired, and has been observed in previous simulation results [14].

B. Case 2a

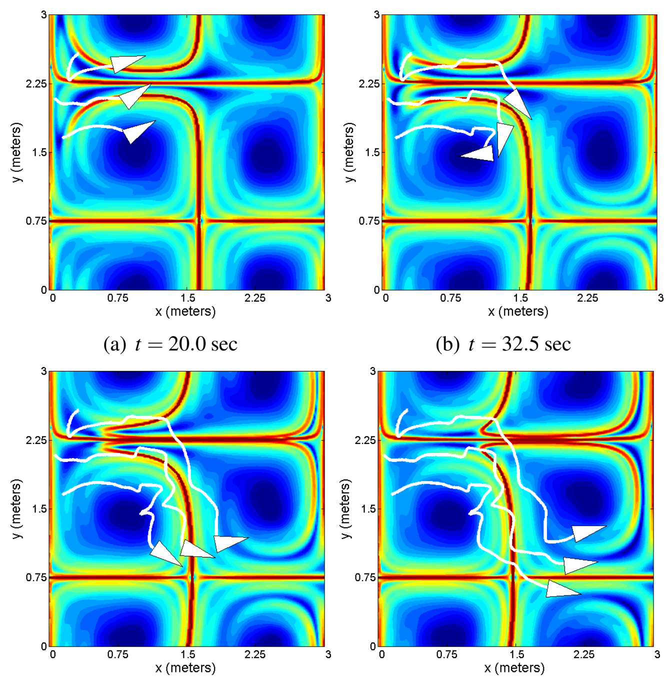
The experiments for Case 2a were performed in still water in the MR tank where the robots' flow sensor outputs were simulated using a *periodic* flow model, i.e., $\mathbf{u}_i$ is given by (2) with $A = 0.2$, $\varepsilon = 0.1$, $\mu = 0.005$, $\omega = 0.05$, $\psi = 0$, and $s = 1.5$. Fig. 5 shows the trajectories of the robot team while tracking the LCS (shown in red) in the flow. The vehicles were able to maintain a straddling formation across the moving LCS as can be seen in the middle of the figure.

[1]The video is also available at `www.pages.drexel.edu/~mam637/IROS2014.mp4`



(a) (b)

Fig. 4. (a) Time invariant double gyre flow field with FLTE field overlayed and (b) paths of a 3 mASVs in MR tank tracking LCS in a simulated time invariant double gyre flow with FTLE field overlayed. Ridges of high FTLE values, indicated by red, define the locations of LCS.



(a) $t = 20.0$ sec (b) $t = 32.5$ sec

(c) $t = 45.0$ sec (d) $t = 57.5$ sec

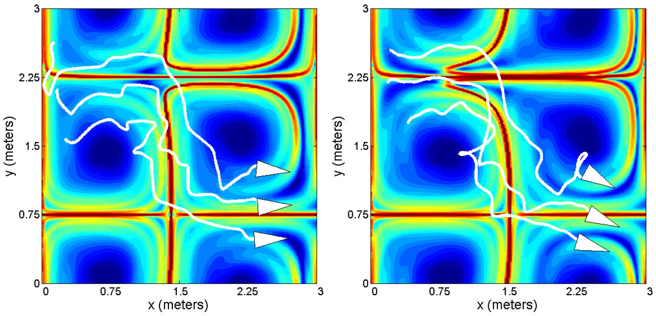
Fig. 5. Paths of 3 mASVs in the MR tank tracking a time varying LCS of a simulated double gyre flow given by (2).

C. Case 2b

The experiments for Case 2b were similar to Case 2a, except zero mean Gaussian noise was added to each $\mathbf{u}_i$ to emulate sensor measurement error. Fig. 6 compares the robot trajectories for runs both with and without sensor noise. Even with a sensor noise standard deviation of 0.14 m/s, which is equal to about 25% of the maximum velocity in the flow, the team is still able to track the time-varying LCS.

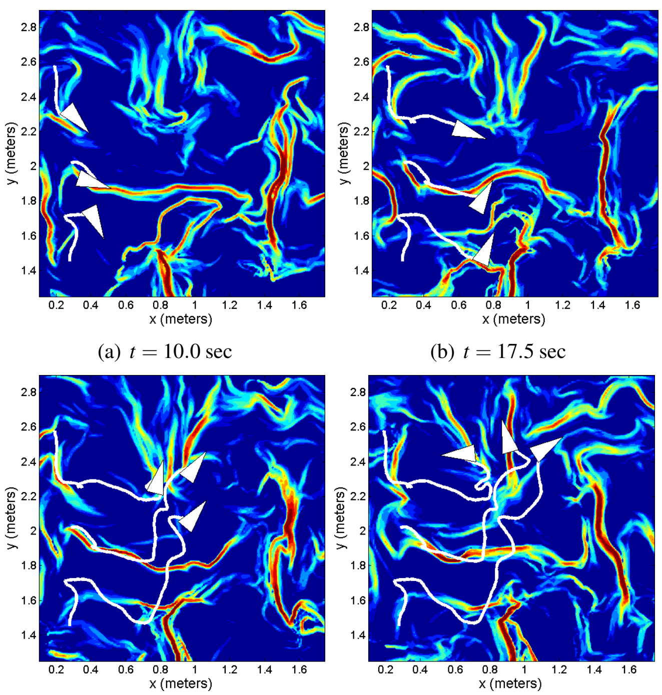
D. Case 3

For this case, a time-invariant flow field was created in the smaller HiRe tank using a grid of 3×4 flow driving cylinders. The data was collected, scaled, and played back during the manifold/LCS tracking experiment. In other words, the



(a) (b)

Fig. 6. Paths of 3 mASVs in the MR tank tracking a time varying LCS of a simulated double gyre flow given by (2) for two different experimental runs. Run (a) was performed without simulated sensor noise, while run (b) was performed with simulated sensor noise. The additive sensor noise was modeled as a zero-mean normal distribution with a standard deviation of 0.14 m/s.



(a) $t = 10.0$ sec (b) $t = 17.5$ sec

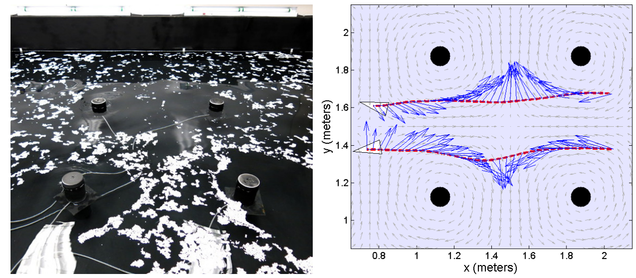
(c) $t = 30.0$ sec (d) $t = 35.0$ sec

Fig. 7. Paths of 3 mASVs in the MR tank tracking a time varying LCS from actual flow data obtained in the High Reynolds number (HiRe) tank.

robots' simulated flow sensor outputs were given by scaled-up actual flow data. Fig. 7 shows that the team tracks an LCS while moving left to right, then after reaching a hyperbolic point in the flow begins to track another LCS while moving upward in the figure.

E. Case 4

In this case, a nearly time invariant multi-gyre flow was generated in the MR tank, and a pair of mASVfs were deployed to sense the flow field while straddling the manifold/LCS between the two gyres. The experimental setup with 4 flow generating cylinders is shown in Fig. 8(a). The trajectories of the two mASVfs operating in the MR tank



(a) (b)

Fig. 8. (a) Four spinning cylinders in the MR tank used to create a gyre like flow and (b) the paths of two mASVfs moving through the flow while sampling the local flow velocity with onboard sensors. The trajectories of the mASVS are shown as dashed red curves and the flow measurements are shown as blue arrows. The intended underlying flow field is shown in gray.

and their flow measurements, obtained with their onboard flow sensors, are shown in Fig. 8(b).

It should be noted that the MR tank was designed to accommodate multiple robots and operate in flows that exhibit the complexity found in actual ocean flows. However, the PIV capabilities of the MR tank are limited. Nonetheless, it is evident the MR tank is approximating the flows we intend based on the data obtained using the HiRe tank. Indeed, the HiRe tank operates in a similar Reynolds number regime with similar flow complexity, but allows for high resolution PIV to validate the flow fields. By using the HiRe tank to guide the MR tank experiments, we can be confident that we are achieving the intended flow dynamics.

V. DISCUSSION

We have previously validated our proposed multi-robot LCS tracking strategy in simulations using analytical models and actual flow data, but challenges arise when implementing the strategy on actual robotic vehicles operating in real flows. In this paper, we have shown that the strategy is amenable for use with actual robotic vehicles as long as the velocities of the robots are larger than the underlying flow velocity. In addition, the flow sensing results from Case 4 (see Sec. IV-E) demonstrate that the mASVfs are capable of reasonably accurate flow velocity sensing of real flows in the MR tank. An implication of this ability to measure flows is that a team of mASVfs can be deployed in the MR tank and track actual LCS in real time. Even with relatively noisy sensors, we have previously shown that the LCS tracking strategy is robust [14]. This shows promise for future field deployments and evaluation of the strategy in actual ocean flows.

VI. ONGOING AND FUTURE WORK

Initial tests using the flows created in both the HiRe and MR tanks have shown that there are significant 3D effects. This is partly due to the manufacturing tolerances of the flow driving cylinder which can cause the cylinders to wobble, creating ripples and waves across the tanks. This not only breaks the 2D assumptions of the flows but also cause significant experimental challenges when controlling the mASV/mASVfs. We are currently investigating alternative designs for the cylinders to reduce these 3D effects. We are also in the process of developing a large scale PIV system to enable global measurements of the flow fields created in the MR tanks. This has proven to be an extremely challenging endeavor due to the large size of the MR tank. Ideally, tracer particles would be chosen such that each individual particle is small enough so as to minimize particle inertial effects, which ensures the measurements reported are of the actual fluid motion. Given the large size of the tank, one challenge is bringing enough pixels to bear to simultaneously provide adequate coverage of the tank while tracking tens of thousands of tiny tracer particles. Second, finding suitable tracer particles has also proven to be difficult since particles tend to aggregate due to surface tension effects, compromising the quality of the PIV measurements. Current efforts are focused on investigating alternative techniques that will enable adequate measurement of the MR tank's surface flows to enable more rigorous analyses of the performance of various robotic experiments.

An immediate direction for future work is to improve the quality of the estimation of the LCS boundaries by increasing the team size and better fusing the various flow measurements obtained using the mASV*f*s' onboard sensors to determine the location of the LCS boundary. While preliminary results show much promise, better increasing the spatial sampling can significantly improve the overall performance of the system. This however can also introduce additional challenges in formation control to ensure the team is able to maintain a desired saddle straddling formation. Another direction for future work is to better understand the impact of the mASV geometry on the local flow and its effects on the onboard flow sensors. This is of particular interest since our tracking strategies depend on the robot's ability to sample the local flow field. By understanding the interactions between the vehicle shape and the surrounding fluid, we can better understand their impact on the quality of the measurements obtained using the onboard flow sensors.

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