

## NON-ZERO ELEMENTS OF THE MATRICES IN EQS. (2) AND (8)

The following notation is used to express the matrix elements:  $Re$  is the Reynolds number,  $Pr$  is the Prandtl number,  $M_e$  is the Mach number at the boundary layer edge and  $g$  is the specific heat ratio. Additionally,  $r = 2(e+2)/3$  and  $m = 2(e-1)/3$  where  $e=0$  corresponds to the Stokes hypothesis.  $U_s(y)$ ,  $W_s(y)$ ,  $T_s(y)$ , and  $\mathbf{m}_s(y)$  are mean-flow profiles. Furthermore,  $\mathbf{a}$  is the streamwise wave number,  $\mathbf{b}$  is the spanwise wave number and  $p$  is the Laplace variable.

We denote  $D = \frac{d}{dy}$  and  $\mathbf{m}'_s = d\mathbf{m}_s / dT_s$ .  $\mathbf{H}^{ij}$  denotes the  $(i, j)$  element of matrix  $\mathbf{H}$ .

$$H_{10}^{21} = Re / \mathbf{m}_s T_s, \quad H_{10}^{34} = -g M_e^2, \quad H_{10}^{35} = 1 / T_s, \quad H_{10}^{43} = -1 / T_s,$$

$$H_{10}^{64} = -(Re Pr / \mathbf{m}_s)(g-1)M_e^2, \quad H_{10}^{65} = Re Pr / T_s \mathbf{m}_s, \quad H_{10}^{87} = Re / T_s \mathbf{m}_s,$$

$$H_{11}^{12} = 1, \quad H_{11}^{22} = -D(\ln \mathbf{m}_s), \quad H_{11}^{23} = (Re / T_s \mathbf{m}_s) D U_s,$$

$$H_{11}^{25} = -D(\mathbf{m}'_s D U_s) / \mathbf{m}_s, \quad H_{11}^{26} = -(\mathbf{m}'_s / \mathbf{m}_s) D U_s, \quad H_{11}^{33} = D T_s / T_s,$$

$$H_{11}^{56} = 1, \quad H_{11}^{62} = -2 D U_s Pr (g-1) M_e^2, \quad H_{11}^{63} = (Re Pr / T_s \mathbf{m}_s) D T_s,$$

$$H_{11}^{68} = -2 Pr D W_s (g-1) M_e^2, \quad H_{11}^{66} = -2 \mathbf{m}'_s D T_s / \mathbf{m}_s,$$

$$H_{11}^{65} = -\left( Pr (g-1) M_e^2 / \mathbf{m}_s \right) \mathbf{m}'_s (D U_s)^2 - D(\mathbf{m}'_s D T_s) / \mathbf{m}_s - \left( Pr (g-1) M_e^2 / \mathbf{m}_s \right) \mathbf{m}'_s (D W_s)^2,$$

$$H_{11}^{78} = 1, \quad H_{11}^{83} = (Re / T_s \mathbf{m}_s) D W_s, \quad H_{11}^{85} = -D(\mathbf{m}'_s D W_s) / \mathbf{m}_s,$$

$$H_{11}^{86} = -(\mathbf{m}'_s / \mathbf{m}_s) D W_s, \quad H_{11}^{88} = -D(\ln \mathbf{m}_s), \quad H_2^{21} = Re U_s / T_s \mathbf{m}_s,$$

$$\begin{aligned}
H_2^{23} &= -D(\ln \mathbf{m}_s), & H_2^{24} &= Re / \mathbf{m}_s, & H_2^{31} &= -1, & H_2^{34} &= -\mathbf{g}M_e^2 U_s, \\
H_2^{35} &= U_s / T_s, & H_2^{41} &= mD\mathbf{m}_s / Re, & H_2^{42} &= (m+1)\mathbf{m}_s / Re, \\
H_2^{43} &= -U_s / T_s, & H_2^{45} &= \mathbf{m}'_s DU_s / Re, & H_2^{63} &= -2PrDU_s(\mathbf{g}-1)M_e^2, \\
H_2^{64} &= -(RePr / \mathbf{m}_s)(\mathbf{g}-1)M_e^2 U_s, & H_2^{65} &= (RePr / T_s \mathbf{m}_s) U_s, \\
H_2^{87} &= ReU_s / T_s \mathbf{m}_s, & H_3^{23} &= -(m+1), & H_4^{21} &= -r, & H_4^{43} &= \mathbf{m}_s / Re, \\
H_4^{65} &= -1, & H_4^{87} &= -1, & L_0^{43} &= -r \mathbf{m}_s / Re, & H_5^{21} &= ReW_s / T_s \mathbf{m}_s, \\
H_5^{34} &= -\mathbf{g}M_e^2 W_s, & H_5^{35} &= W_s / T_s, & H_5^{37} &= -1, & H_5^{43} &= -W_s / T_s, \\
H_5^{45} &= \mathbf{m}'_s DW_s / Re, & H_5^{47} &= mD\mathbf{m}_s / Re, & H_5^{48} &= (m+1)\mathbf{m}_s / Re, \\
H_5^{63} &= -2PrDW_s(\mathbf{g}-1)M_e^2, & H_5^{64} &= -(RePr / \mathbf{m}_s)(\mathbf{g}-1)M_e^2 W_s, \\
H_5^{65} &= (RePr / T_s \mathbf{m}_s) W_s, & H_5^{83} &= -D(\ln \mathbf{m}_s), & H_5^{84} &= Re / \mathbf{m}_s, \\
H_5^{87} &= ReW_s / T_s \mathbf{m}_s, & H_6^{27} &= -(m+1), & H_6^{81} &= -(m+1), \\
H_7^{83} &= -(m+1), & H_8^{21} &= -1, & H_8^{65} &= -1, & H_8^{87} &= -r, & H_8^{43} &= \mathbf{m}_s / Re, \\
H_0^{12} &= 1, & H_0^{21} &= \mathbf{a}^2 + \mathbf{b}^2 + i(\mathbf{a}U_s + \mathbf{b}W_s - ip)Re / \mathbf{m}_s T_s, \\
H_0^{22} &= -D\mathbf{m}_s / \mathbf{m}_s, & H_0^{23} &= -i\mathbf{a}(m+1)DT_s / T_s - i\mathbf{a}D\mathbf{m}_s / \mathbf{m}_s + ReDU_s / \mathbf{m}_s T_s, \\
H_0^{24} &= i\mathbf{a}Re / \mathbf{m}_s - (m+1)\mathbf{g}M_e^2 \mathbf{a}(\mathbf{a}U_s + \mathbf{b}W_s - ip), \\
H_0^{25} &= \mathbf{a}(m+1)(\mathbf{a}U_s + \mathbf{b}W_s - ip) / T_s - D(\mathbf{m}'_s DU_s) / \mathbf{m}_s, \\
H_0^{26} &= -\mathbf{m}'_s DU_s / \mathbf{m}_s, & H_0^{31} &= -i\mathbf{a}, & H_0^{33} &= DT_s / T_s, \\
H_0^{34} &= -i\mathbf{g}M_e^2 (\mathbf{a}U_s + \mathbf{b}W_s - ip), & H_0^{35} &= i(\mathbf{a}U_s + \mathbf{b}W_s - ip) / T_s,
\end{aligned}$$

$$H_0^{37} = -i\mathbf{b}, \quad \mathbf{c} = \left[ \text{Re}/\mathbf{m}_s + ir\mathbf{g}M_e^2(\mathbf{a}U_s + \mathbf{b}W_s - ip) \right]^{-1},$$

$$H_0^{41} = -iac(rDT_s/T_s + 2D\mathbf{m}_s/\mathbf{m}_s), \quad H_0^{42} = -ica,$$

$$H_0^{43} = \mathbf{c} \left[ -\mathbf{a}^2 - \mathbf{b}^2 - i(\mathbf{a}U_s + \mathbf{b}W_s - ip)\text{Re}/\mathbf{m}_sT_s + rD^2T_s/T_s + rD\mathbf{m}_sDT_s/\mathbf{m}_sT_s \right],$$

$$H_0^{44} = -icr\mathbf{g}M_e^2 \left[ \mathbf{a}DU_s + \mathbf{b}DW_s + (\mathbf{a}U_s + \mathbf{b}W_s - ip)(DT_s/T_s + D\mathbf{m}_s/\mathbf{m}_s) \right],$$

$$H_0^{45} = ic \left[ (\mathbf{a}DU_s + \mathbf{b}DW_s)(r/T_s + \mathbf{m}'_s/\mathbf{m}_s) + r(\mathbf{a}U_s + \mathbf{b}W_s - ip)D\mathbf{m}_s/\mathbf{m}_sT_s \right],$$

$$H_0^{46} = irc(\mathbf{a}U_s + \mathbf{b}W_s - ip)/T_s, \quad H_0^{47} = -i\mathbf{bc}(rDT_s/T_s + 2D\mathbf{m}_s/\mathbf{m}_s),$$

$$H_0^{48} = -i\mathbf{bc}, \quad H_0^{56} = 1, \quad H_0^{62} = -2(\mathbf{g}-1)M_e^2PrDU_s,$$

$$H_0^{63} = -2i(\mathbf{g}-1)M_e^2Pr(\mathbf{a}DU_s + \mathbf{b}DW_s) + \text{Re}PrDT_s/\mathbf{m}_sT_s,$$

$$H_0^{64} = -i\text{Re}Pr(\mathbf{g}-1)M_e^2(\mathbf{a}U_s + \mathbf{b}W_s - ip)/\mathbf{m}_s,$$

$$H_0^{65} = \mathbf{a}^2 + \mathbf{b}^2 + i\text{Re}Pr(\mathbf{a}U_s + \mathbf{b}W_s - ip)/\mathbf{m}_sT_s - (\mathbf{g}-1)M_e^2Pr\mathbf{m}'_s \left[ (DU_s)^2 + (DW_s)^2 \right] / \mathbf{m}_s - D^2\mathbf{m}_s/\mathbf{m}_s,$$

$$H_0^{66} = -2D\mathbf{m}_s/\mathbf{m}_s, \quad H_0^{68} = -2(\mathbf{g}-1)M_e^2PrDW_s, \quad H_0^{78} = 1,$$

$$H_0^{83} = -i(m+1)\mathbf{b}DT_s/T_s - i\mathbf{b}D\mathbf{m}_s/\mathbf{m}_s + \text{Re}DW_s/\mathbf{m}_sT_s,$$

$$H_0^{84} = -(m+1)\mathbf{g}M_e^2\mathbf{b}(\mathbf{a}U_s + \mathbf{b}W_s - ip) + i\mathbf{b}\text{Re}/\mathbf{m}_s,$$

$$H_0^{85} = (m+1)\mathbf{b}(\mathbf{a}U_s + \mathbf{b}W_s - ip)/T_s - D(\mathbf{m}'_sDW_s)/\mathbf{m}_s,$$

$$H_0^{86} = -\mathbf{m}'_sDW_s/\mathbf{m}_s, \quad H_0^{87} = \mathbf{a}^2 + \mathbf{b}^2 + i\text{Re}(\mathbf{a}U_s + \mathbf{b}W_s - ip)/\mathbf{m}_sT_s,$$

$$H_0^{88} = -D\mathbf{m}_s/\mathbf{m}_s$$